I. PHYSICAL ELECTRONICS

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RESEARCH OBJECTIVES

Present objectives lie in three separate areas of physical electronics. The first relates to the properties of semiconductors, the second to thermionic emission problems, and the third to the improvement of instrumentation.

The electrical properties of p-n junctions are being investigated as a function of the temperature and the applied voltage. The observations include a careful determination of the current flow as a function of the voltage and also of the capacity of the junction as a function of the voltage. Analysis of such observations yields interesting data, the interpretation of which will serve to measure the applicability of present theories to the electrical properties of junctions.

Not only is it of interest to study the properties of semiconductors, but we consider it a worth-while objective to use semiconducting devices in our instrumentation. One such development is the design and construction of a micro-microammeter that is capable of measuring currents with an accuracy of 1 per cent, or better, over the entire range from 1 ma down to 10^{-14}. This device uses a combination of vacuum tubes and solid-state rectifiers and transistors. Our development is aimed, specifically, toward perfecting the stability of the unit with respect to power-line variations and room-temperature changes. A model of this micro-microammeter is being used regularly in this laboratory, in order to determine features that need improvement before a detailed description of it is published.

The thermionic studies now in progress relate in one way or another to the direct conversion of heat to electric power through the use of thermionic emitters. Three phases are now being studied. One is the energy distribution of electrons emitted from a hollow cathode under high-vacuum conditions. The purpose of this study is to determine whether or not electrons emitted under these circumstances take on an abnormal energy distribution that is not characterized by the temperature of the emitter. A second study is designed to determine the adsorption properties of well-defined crystallographic directions of tungsten for cesium as a function of the cesium pressure and the tungsten temperature. The third phase of this study is an attempt to interpret experimental data by a theoretical analysis that is dependent upon the fundamentals of physical electronics.

W. B. Nottingham

A. PHYSICAL ELECTRONICS IN THE SOLID STATE

1. CHARACTERISTICS OF SEMICONDUCTOR JUNCTIONS

In Quarterly Progress Report No. 55, pages 1-3, the forward characteristics of two junctions were given, and it was shown that in the lower portion of the characteristic the current is well represented by an equation of the form $I \alpha \exp(BV)$, where B is independent of temperature.

In an effort to pin down the specific mechanism for this somewhat anomalous characteristic, some studies of the time-dependence of the characteristic are being carried out. In most of the junctions studied thus far, the reverse characteristics have been somewhat unstable. Although this effect is definitely reproducible from day to day, during any run we find that the current for an applied voltage slowly increases or decreases with time. In a particular junction it was found that the current decays for a certain range of voltages, and increases for another, higher, range of voltages. By following this decay with a recorder, we have found that the decay (or increase) is approximately exponential in time, and that the time constant is highly temperature-dependent. The relation of these changes to surface states and channel formation is still not clear, but this explanation is the most likely one.

J. F. Campbell, Jr.

B. ELECTRON EMISSION AND CESIUM PLASMA

1. THERMIONIC ENERGY CONVERTERS

Two types of thermionic energy converters fall under the general classifications of vacuum diode and plasma diode. Recent calculations have been carried through, applicable to each of these, and this report will cover, first, the recent additions relative to the vacuum diode, and second, some ideas relevant to the plasma diode.

a. Vacuum Diode

My first paper on this subject (1) discussed some of the basic features of the plane parallel vacuum diode and derived equations by which their properties could be predicted from a knowledge of the emitter temperature, the spacing, and the receiver work-function. At the time when this paper was written, I did not consider that diodes built with a spacing of less than 25μ would be of much practical interest. Under this condition, the work-function of the emitter plays a relatively small part, and therefore the final equations put forward were limited in their scope. This fact was pointed out to me by Dr. E. S. Rittner, and new equations that may be applied to very closely spaced diodes with cathodes of limited emission capability have now been developed. Although it is anticipated that a more complete report will be made, the present one will indicate the new approach as concisely as possible.

b. General Outline of Procedure

Important symbols that are needed for this discussion are illustrated graphically in Fig. I-1. This figure shows the electron motive diagram for a diode of actual spacing w, from the Fermi level of the emitter to the work-function barrier ϕ_1 just at the surface of the emitter, across the evacuated space to the surface of the collector, and then inside the collector to its Fermi level. The controllable voltage V has been adjusted to V_R so that the space-charge distribution of the electrons in transit results in zero field coinciding with the surface of the collector. Under this condition, the current density carried from the emitter to the collector is I_R. The following equation gives

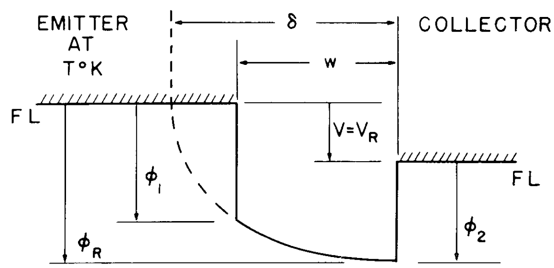

Fig. I-1. Electron potential distribution with critical condition of zero gradient at the collector.

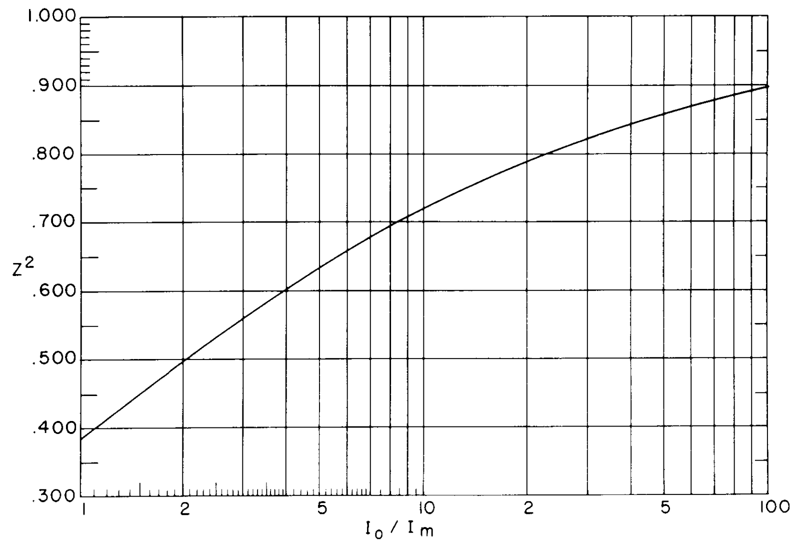

Fig. I-2. Computational chart relating z^2 to (I_o/I_m).

the absolute maximum value of electron current I_m that can flow across a diode of spacing w from an emitter at a temperature T, under the condition of zero field at the collector:

$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \tag{1}$$

We define

$$V_T = \frac{kT}{q} = 11,600 \tag{2}$$

In Eq. 1 the current density will be in amp/cm^2 if the distance w is expressed in centimeters. The true value of I_R is always less than I_m because the actual potential distribution in this space is that associated with a diode of augmented spacing that is shown in Fig. I-1 as δ. The following equation relates these quantities and defines a new parameter, z:

$$\frac{w^2}{\delta^2} = z^2 = \frac{I_R}{I_m} \tag{3}$$

The true zero-field emission capability of a cathode is determined by its temperature and the value of the true work-function ϕ_1. This emission can be computed accurately by using

$$I_o = 120\ T^2 \exp(-\phi_1/V_T) \tag{4}$$

These quantities may be related by

$$\frac{I_o}{I_m} = z^2 \exp[(\phi_R - \phi_1)/V_T] \tag{5}$$

The Langmuir space-charge theory applied to the relation in Eq. 5, as shown in Section 43 of "Thermionic Emission" (2), permits the computation of the correct value of z^2 as a function of (I_o/I_m). A table of values is given there and the graphical presentation of this table is shown in Fig. I-2. With z^2 known, the value of I_R is given by Eq. 3.

c. Procedure for the Computation of Diode Properties

Step 1: Compute the ratio (I_o/I_m) from Eqs. 1 and 4, or directly by means of Eq. 6.

$$\frac{I_o}{I_m} = 1.558 \times 10^{13}\ w^2\ T^{1/2} \exp(-\phi_1/V_T) \tag{6}$$

Step 2: Refer to Table 4 of "Thermionic Emission," or to the curve of Fig. I-2 to determine z^2. For very large values of (I_o/I_m), the following equation may be used:

$$z^2 = 1 - \frac{1.11}{\left(\frac{I_o}{I_m}\right)^{1/2}} \tag{7}$$

Step 3: The value of V_R (see Fig. I-1) can be computed as

$$V_R = \phi_1 + V_T \ln \frac{I_o}{I_R} - \phi_2 \tag{8}$$

Step 4: Evaluate the ratio (V_R/V_T). Either one of two procedures that depend on the value of this ratio are needed.

For Case I, we have

$$1 < \frac{V_R}{V_T} < 12 \tag{9}$$

Figure I-3 shows schematically the motive diagram for the diode delivering maximum power. If V is the voltmeter reading that indicates the separation between the

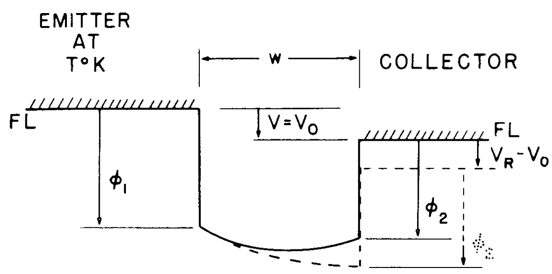

Fig. I-3. Potential distribution with maximum power in load.

Fermi level of the emitter and the Fermi level of the collector, there will be a particular value of V, namely V_O, for which the maximum power can be delivered to an external load. The following equation defines a symbol used in Table 8 of "Thermionic Emission":

$$S' = \Sigma = (V_R - V)/V_T \tag{10}$$

The value of this parameter under the condition of maximum power is

$$S'_{max} = \Sigma_{max} = (V_R - V_O)/V_T \tag{11}$$

subject to the condition that $I_{max} < 0.38\ I_o$, where I_{max} is the current at maximum power output. A detailed study has been made for various values of the ratio (V_R/V_T), and the following relation serves as an accurate means of determining the value of Σ at the maximum power:

$$\Sigma_{max} = 0.556\left(\frac{V_R}{V_T} - 1\right) \tag{12}$$

Corresponding to each value of Σ there is a value of U^2 listed in Table 8. The quantity U^2 is defined as

$$U^2 = \frac{I_\infty}{I_m} \tag{13}$$

With I_R and I_o known, emission capability can be expressed as

$$u_o^2 = \frac{I_o}{I_R} \tag{14}$$

Emission demand can be expressed as

$$u^2 = \frac{I}{I_R} \tag{15}$$

For practically all diodes,

$$u^2 = U^2 \tag{16}$$

with a 1 per cent error as u^2 approaches the value $0.38\ u_o^2$. Even if the demand is greater than $0.38\ u_o^2$, the problem can be solved. It follows that the current at maximum power is given by

$$I_{max} = I_R\ U_{max}^2 \tag{17}$$

Although U_{max}^2 can be determined from an accurate plotting of the data of Table 8, it is more convenient to use the following empirical equation:

$$U_{max}^2 = 1 + \Sigma_{max} + c(\Sigma_{max})^m \tag{18}$$

In the range of Σ_{max}, 0.5-5: $c = 0.08$, $m = 1.85$; in range 4-15: $c = 0.1$, $m = 1.7$; in range 10-100: $c = 0.11$, $m = 1.65$. The current at maximum power output can now be determined from Eq. 17. The voltage output can be determined from Eq. 11 and rearranged as follows:

$$V_O = V_T\left(\frac{V_R}{V_T} - \Sigma_{max}\right) \tag{19}$$

The maximum power output is the product expressed as follows:

$$P_{max} = V_O\ I_{max} = I_R\ U_{max}^2\left(\frac{V_R}{V_T} - \Sigma_{max}\right) V_T \tag{20}$$

For Case II, we have

$$6 < \frac{V_R}{V_T} < 20 \tag{21}$$

Equations previously published (1) serve as a means of computing the maximum power from the output voltage and current density. These equations, modified by the use of I_R instead of I_m, are written

$$I_{max} = I_R \left[1 + 0.31 \left(\frac{V_R}{V_T} \right)^{4/3} \right] \tag{22}$$

$$V_O = \frac{0.383 \left(\frac{V_R}{V_T} \right) V_R}{1 + 0.31 \left(\frac{V_R}{V_T} \right)^{4/3}} \tag{23}$$

$$P_{max} = 0.383 \left(\frac{V_R}{V_T} \right) V_R \, I_R \tag{24}$$

Under extreme conditions of close spacing and limited cathode activity, the value of U^2_{max} calculated by Eq. 18 may be greater than $0.38\ u_o^2$ calculated by Eq. 14. In that case the data covered by Table 9 of "Thermionic Emission" (2) will provide the means of making an additional correction, and thus provide a more accurate value of the current obtainable at maximum power from such a diode. The equations given here will be found more satisfactory than those published previously for the close-spaced diode with a limited emission capability at the cathode. The work-function of the cathode must be known with reasonable accuracy for correct design calculations (3).

d. Cesium Plasma Diode as an Energy Converter

There are two classifications of plasma diodes which are likely to be practical. Relatively low pressure cesium diodes may be expected to be of interest in connection with low work-function emitters and very moderate spacings. Such diodes will probably be plane diodes, or will have cylindrical electrodes with a difference in radii which is small compared with the radius of either the emitter or the collector. The high-pressure diode can be expected to be effective with high-temperature emitters and a large enough spacing so that the mean-free path for electrons in the cesium vapor will not be long compared with the spacing. In fact, we might anticipate a spacing of approximately 10-30 mean-free paths. If it is possible, from a design point of view, to have a spacing 3 or 4 times larger than the emitter radius, that will also probably be an advantage.

Many of the equations useful for the analysis of the plasma diode have been published (4). The center of attention in this published material relates mainly to the low-pressure diodes, although it is not exclusively in that area. In the low-pressure diode, the electron-current flow is likely to be limited by an electron space charge that is very close to the emitter, and the influence of the cesium ions that are ionized at the emitter surface will, in general, be to reduce the space charge, although there may not be enough ions to eliminate it. As the ion production increases, it may create a region of <u>positive</u> <u>ion</u> space charge at the emitter surface. The present remarks are addressed

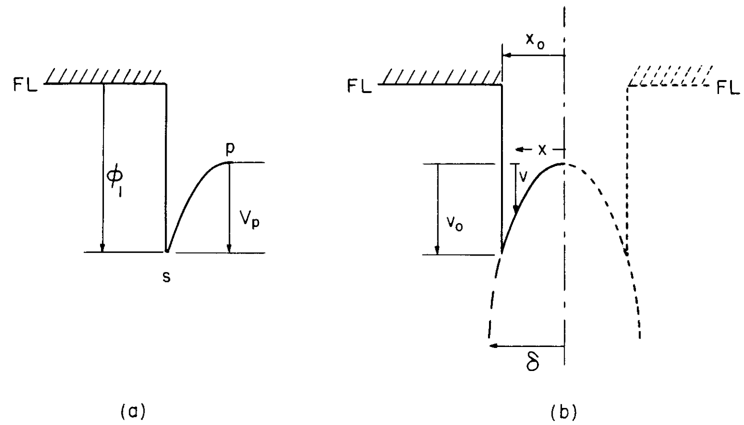

Fig. I-4. Positive ion space charge: (a) at emitter surface (plasma at p); (b) in a cavity.

specifically to the discussion of factors of particular importance when the ion density is so high that a strong, positive ion space charge exists at the emitter surface.

If the true work-function of the surface is something like 4 or 5 electron volts, and the temperature sufficiently high (2100°K-2800°K), then every neutral cesium atom that strikes the surface can be expected to become ionized. It is possible to estimate the thickness of this sheet which is defined here as the distance between the hot surface and the point in space at which the ion density equals the electron density. This condition is illustrated by Fig. I-4a.

It is of interest to attempt to solve Poisson's equation with a Maxwellian distribution in energy of the ions in the space-charge sheath. This problem can be worked out precisely as Fowler did for the potential distribution across a cavity containing charged particles. This distribution is shown in Fig. I-4b, with half of the motive pattern dotted and the other half solid, to illustrate, at least qualitatively, the similarity between the cavity space-charge problem and the sheath problem. Sections 22, 23, and 24 of "Thermionic Emission" (2) fill in details that are omitted here. In relation to Fig. I-4b, Eq. 25 expresses the ion density at any point between the surface and the midpoint of the cavity. This is expressed in terms of the motive potential relative to that at the midpoint, and the coefficient n_o is the ion density at the midpoint.

$$n_+ = n_o \exp(qv/kT) \tag{25}$$

The field at the midpoint is zero because of the negative surface charge on each of these surfaces. In the solution for the sheath problem, the point of zero field is an indication that the ion density is exactly equal to the electron density.

The solution to Poisson's equation yields the following relation between the change in motive potential v and the distance x from the midpoint.

$$x = \left[\frac{2kT\,\epsilon_o}{q^2 n_o}\right]^{1/2} \arctan\left(e^{(qv/kT)}-1\right)^{1/2} \tag{26}$$

The electric intensity at any point at which the potential is v is given by:

$$E_x = \left(\frac{2kT\,n_o}{\epsilon_o}\right)^{1/2} \left(e^{(qv/kT)}-1\right)^{1/2} \tag{27}$$

An equation that is useful in the analysis serves to relate δ to n_{om} as follows:

$$\delta^2 = \frac{kT\,\epsilon_o}{q^2 n_{om}} \frac{\pi^2}{2} \tag{28}$$

This equation means that for any center density n_{om}, there is a maximum value of distance δ which cannot be exceeded no matter how effective the surface is as an ion source for building up the space charge. In any specific case, the true surface will be at a distance x_o from the center line and the following relation serves to define a parameter z^2.

$$z^2 = \frac{x_o^2}{\delta^2} \tag{29}$$

$$n_o = z^2 n_{om} \tag{30}$$

From Eq. 28 a maximum possible value, n_{om}, can be related to the spacing and the temperature if the δ^2 used there is set equal to the spacing factor, x_o^2. If the concentration of ions at the surface is identified as n_s, then a functional relation exists between z^2 and (n_s/n_{om}). This functional relation is recorded in Table 1 of "Thermionic Emission" (2).

If the work-function of the surface ϕ_1 is equal to or greater than the ionization potential of cesium, an analysis that applies to an ion sheath in a cesium-filled diode shows that the ion density (ions/cm^3) at the surface can be computed by Eq. 31.

$$\ln n_s = 66.04 - \frac{8980}{T_{Cs}} + \frac{\phi_1 - V_i}{V_T} - \frac{1}{2}\ln T \tag{31}$$

Here T_{Cs} is the temperature of the cesium source in equilibrium with liquid cesium; V_i is the ionization potential of 3.88 volts. If this analysis is correct, then the entire problem of the distribution in potential and the intensity of field at the surface can be solved. Clearly, this is an idealization because it neglects, to some extent, the detailed influence of the electrons within the sheath.

We can estimate the surface concentration of ions (n_s), and also the ion concentration (n_o) at the point p in the plasma that is illustrated in Fig. I-4a. If these two values can be specified, they may be used in Table 1 of "Thermionic Emission" to determine a value of z^2 that is suitable for use. An equation by means of which the value of (V_p/V_T) can be determined is

$$\frac{V_p}{V_T} - \frac{1}{2}\ln\left(\frac{V_p}{V_T}+1\right) = 28.07 - \frac{8980}{T_{Cs}} - \frac{V_i}{V_T} + \frac{2\phi_1}{V_T} - \frac{5}{2}\ln T - \frac{1}{2}\ln V_T^{-1} + \ln(1-f_o) \tag{32}$$

The fractional ionization at the sheath boundary is f_o.

Figure I-5 has been prepared as an aid to the determination of (V_p/V_T). The right-hand side of Eq. 32 may be computed in terms of T_{Cs}, T, V_T, and ϕ_1 and serves to define $f(V_P/V_T)$. For preliminary calculations assume f_o to be approximately 0.5 at the sheath boundary even though the fractional ionization may be higher in the plasma proper.

The thermionic emission current density can be expressed as follows:

$$I = 120\ T^2 \exp(-\phi_1/V_T) \tag{33}$$

The electron density, which is taken to be equal to the ion density at p of Fig. I-4a, is given by

$$n_o = \frac{I}{q}\left[\frac{m}{2q(V_p+V_T)}\right]^{1/2} 10^{-2} \tag{34a}$$

$$n_o = 1.053 \times 10^{11} \frac{I}{(V_p+V_T)^{1/2}} \tag{34b}$$

To realize a specific number from this equation, the value of V_p must have been determined from Eq. 32.

Once suitable values of these factors have been chosen, Eq. 26 may be used to estimate the sheath thickness, and Eq. 27 may be used to estimate the surface field associated with this solution of the space-charge equation. Knowing the surface field, one can roughly evaluate the reduction in average work-function resulting from this field. It is to be anticipated that even more electrons will become available than might have been expected on the basis of a known true work-function of the emitter reduced by a Schottky field reduction because of the localized influence of individual ions.

These results will be illustrated by a numerical example. Assume that V_T is 0.2, and the corresponding temperature is T = 2320°K. If the value of ϕ_1 is 4.0 ev, and the cesium temperature T_{Cs} is 500°K, then the value of V_p is 2.2 volts.

This calculation indicates that electrons will be injected across the ion space-charge sheath, which is short compared with the mean-free path, with an average total energy

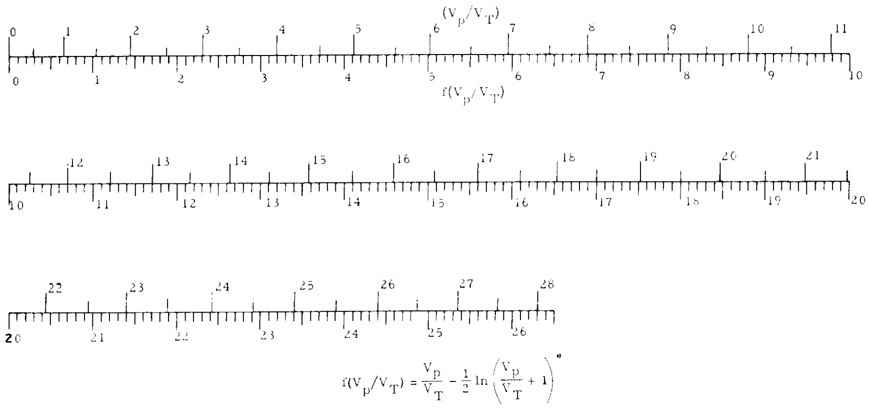

Fig. I-5. Correlation chart of (V_p/V_T) to $f(V_p/V_T)$.

per electron of approximately 2.6 ev. It is to be anticipated that high-frequency plasma oscillations will develop if the spacing is not too short. A redistribution of the electron energy will take place from a distribution that was more or less "mono-energetic" to one that may be characterized roughly by a temperature of 20,000°K. Such a redistribution in energy can take place in the presence of oscillations without the addition of any total energy in the space because the average energy associated with such a distribution remains unchanged. There is a net gain, however, in that with this redistribution in energy there are now many electrons available for the creation of new ions in the space. In this way, a plasma is developed, even though no accelerating field exists in the space to give additional energy to the electrons.

These remarks and equations are presented in this preliminary form to indicate a direction that seems likely to be fruitful in the interpretation of the high-pressure plasma diode designed for the purpose of the direct conversion of heat-to-electric power.

W. B. Nottingham

References

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2. W. B. Nottingham, Thermionic emission, Handbuch der Physik, Vol. 21 (Springer Verlag, Berlin-Göttingen-Heidelberg, 1956), p. 1; Technical Report 321, Research Laboratory of Electronics, M.I.T., Dec. 10, 1956.

3. A detailed paper covering this subject matter has been submitted for publication to the Journal of Applied Physics.

4. W. B. Nottingham, Cesium plasma diode as a heat-to-electrical power transducer, paper presented at Fourth International Conference on Ionization Phenomena, Uppsala, Sweden, 1959; Cesium plasma diode as a heat-to-electrical-power transducer, Chap. 8, p. 1, and The thermionic diode as a heat-to-electrical-power transducer, Chap. 2, p. 1, in Direct Conversion of Heat to Electricity, Summer Program Course, Department of Mechanical Engineering, M.I.T., 1959.